Holomorphic approximation theory

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In this talk we discuss basic holomorphic approximation theory. We begin with the theorem by C. Runge from 1885 stating that given a compact set $K \subset \mathbb{C}$ such that its complement $\mathbb{C} \setminus K$ is connected, any function, holomorphic on a neighbourhood of K, is a uniform limit on K of a sequence of holomorphic polynomials. We prove the theorem as well as a generalized version for an arbitrary compact set in $\mathbb{C}$. Among various applications of Runge's result is the Mittag-Leffler's theorem concerning meromorphic functions with prescribed principal parts at a given closed discrete set. It was proven in 1949 that the same holds on every open Riemann surface. We mention some other extensions, which have been of particular interest in the recent years.

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