

Talks in the ESDG

First week

Monday 24

Speaker: Gabriella Tarantello (University of Rome Tor Vergata)

Title: On CMC-immersions of surfaces into Hyperbolic 3-manifolds.

Abstract: I shall discuss the moduli space of Constant Mean Curvature (CMC) c -immersions of a closed surface S (orientable and of genus at least 2) into hyperbolic 3-manifolds. Interestingly when $|c| < 1$, such space admits a nice parametrization described by elements of the tangent bundle of the Teichmueller space of S . Indeed, for any such element we shall see how to determine uniquely the pullback metric and the second fundamental form of the immersion by solving the “constrained” Gauss-Codazzi equations. This is attained by showing that the associated action functional (known as the “Donaldson-functional” in Gonsalves-Uhlenbeck (2007)) admits a global minimum as its unique critical point.

In addition I shall discuss the asymptotic behavior of those minimizers as $|c|$ approaches 1 and see when it is possible to obtain “convergence” to a (CMC) 1-immersion. Note that (CMC) 1-immersions into the hyperbolic space are particularly relevant in hyperbolic geometry in view of their analogies with minimal immersions into the Euclidean space.

For example, we show that for genus 2, it is possible to catch at the limit a “regular” CMC 1-immersion into an hyperbolic 3-manifold, except in very rare situations which relate to the image, under the Kodaira map, of the six Weierstrass points of S . Finally, I shall discuss further progress for higher genus obtained in collaboration with S. Trapani.

Speaker: M. Traizet (University of Tours)

Title: Loop group methods for harmonic maps into the 3-sphere and hyperbolic 3-space.

Abstract: I will explain Hitchin's gauge theoretic formulation of the harmonic map equation from a Riemann surface to \mathbb{S}^3 or \mathbb{H}^3 , in terms of families of flat connections on a fibre bundle.

Then I will explain how the DPW method (or generalized Weierstrass representation) is best understood as a recipe to construct such families of connections from holomorphic data, using a loop group factorisation.

These loop group methods allow for quite explicit computations. I will give applications to Lawson minimal surfaces in the spherical case, and to the non-abelian Hodge correspondence in the hyperbolic case.

Joint work with L. Heller and S. Heller.

Tuesday 25

Speaker: Claude LeBrun (Stony Brook University)

Title: Einstein Metrics, 4-Manifolds, and Gravitational Instantons.

Abstract: A Riemannian metric is said to be **Einstein** if it has constant Ricci curvature. Certain peculiar features of 4-dimensional geometry make dimension four into a "Goldilocks zone" for Einstein metrics, with just the right amount of local flexibility managing to coexist with strong global rigidity results. This talk will first describe some aspects of the interplay between Einstein metrics and smooth topology on compact symplectic 4-manifolds without boundary. We will see how ideas from Kähler and conformal geometry allow us to construct Einstein metrics on many such manifolds, while a complimentary tool-box shows that these existence results are optimal in certain specific contexts. The talk will then conclude with a brief discussion of analogous results concerning complete Ricci-flat 4-manifolds.

Speaker: Ana Menezes (Princeton University)

Title: Eigenvalue problems and free boundary minimal surfaces in spherical caps.

Abstract: In a joint work with Vanderson Lima (UFRGS, Brazil), we introduced a family of functionals on the space of Riemannian metrics of a compact surface with boundary, defined via eigenvalues of a Steklov-type problem. In this talk we will prove that each such functional is uniformly bounded from above, and we will characterize maximizing metrics as induced by free boundary minimal immersions in some geodesic ball of a round sphere.

Also, we will determine that the maximizer in the case of a disk is a spherical cap of dimension two, and we will prove rotational symmetry of free boundary minimal annuli in geodesic balls of round spheres which are immersed by first eigenfunctions.

Wednesday 26

Speaker: Theodoros Vlachos (University of Ioannina)

Title: Ricci pinched compact submanifolds in space forms.

Abstract: We investigate compact submanifolds in Riemannian space forms of nonnegative sectional curvature that satisfy a lower bound on the Ricci curvature, that bound depending solely on the length of the mean curvature vector of the immersion. Just in special cases, the limited strength of the assumption allows some strong additional information on the extrinsic geometry of the submanifold. While generalizing the results, we give a positive answer to a conjecture by H. Xu and J. Gu in (2013, *Geom. Funct. Anal.* 23). Our main accomplishment is the elimination of the need for the mean curvature vector field to be parallel. The results are joint work with Marcos Dajczer.

Speaker: Andrea Seppi (Université Grenoble Alpes)

Title: Uniqueness and non-uniqueness for the Asymptotic Plateau Problem in hyperbolic three-space.

Abstract: The Asymptotic Plateau Problem in the hyperbolic space is the problem of existence of minimal surfaces with a prescribed Jordan curve as a boundary “at infinity”. Since the work of Anderson in the 1980s, it is known to have a solution, which is however in general not unique. In this talk, I will give an overview of the subject, present examples of “pathological” non-uniqueness, and describe some criteria for uniqueness.

Thursday 27

Speaker: Felix Schulze (Warwick University)

Title: Mean Curvature Flow from conical singularities.

Abstract: We give a proof of Ilmanen’s resolution of point singularities conjecture by establishing short-time smoothness of the level set flow of a smooth hypersurface with isolated conical singularities. Combined with the

uniqueness of asymptotically conical tangent flows, this shows how the outermost mean curvature flows evolve through such singularities and how mean curvature flow becomes non-unique past such singularities. Furthermore, we resolve a particular case of Ilmanen’s strict genus reduction conjecture. Precisely, we prove that the level set flow of a smooth hypersurface $M^n \subset \mathbb{R}^{n+1}$, $2 \leq n \leq 6$, with an isolated conical singularity is modelled on the level set flow of the cone. In particular, the flow fattens (instantaneously) if and only if the level set flow of the cone fattens. This is joint work with Otis Chodosh and Joshua Daniels-Holgate.

Speaker: Theodora Bourni (University of Tennessee)

Title: Constructing solution to curve shortening and related flows.

Abstract: We will discuss the construction of certain interesting solutions to curve shortening and related flows. Some of these lead to classification results for ancient solutions.

Friday 28

Speaker: Laurent Mazet (University of Tours)

Title: Stable minimal hypersurfaces in \mathbb{R}^6 .

Abstract: The stable Bernstein problem asks whether a stable minimal hypersurface in \mathbb{R}^{n+1} is a Euclidean hyperplane. The answer is known to be no if $n \geq 7$. In this talk, I want to explain the elements that leads to a positive answer when $n = 5$ and the hypersurface is two-sided.

Speaker: Roman Petrides (Université Paris Diderot)

Title: Spectral optimization on a surface whatever its topology.

Abstract: We prove that the maximization of the first eigenvalue of the Laplacian with respect to Riemannian metrics of fixed area is realized on a closed surface whatever its topology. The answer to this question was only known for topologies with low genres and was open since the seminal papers of Hersch 1970 (sphere), Li-Yau 1982 (projective plane), Berger 1973, Nadirashvili 1996 (tori).

Our result relies on constructions and convergence properties of accurate maximizing sequences that satisfy a Palais-Smale-like condition on the locally Lipschitz functional “first eigenvalue”. We will also explain why the optimization result also holds for a wide class of combinations of Laplace eigenvalues and what analogously happens for the optimization of Steklov

eigenvalues on surfaces with boundary.

Second week

Monday 1

Speaker: Antonio Ros (University of Granada)

Title: Minimal surfaces and the first eigenvalue of the Laplacian

Abstract: Given a closed Riemann Surface, the normalized first eigenvalue of the Laplacian is a natural functional defined in the space of conformal Riemannian metrics. Critical points, in particular maxima, their relations to conformal geometry and minimal surfaces of the sphere constitute an interesting and active field of research. We will consider these issues by focusing in Riemann surfaces with large conformal group.

Applications will be given to the cases of Bolza's surface of genus 2 and Klein's quartic of genus 3.

Speaker: Alexander Bobenko (Institut für Mathematik, TU Berlin)

Title: The Bonnet problem: Is a surface characterized by its metric and curvatures?

Abstract: A longstanding problem in differential geometry, known as the Bonnet problem, is whether a compact surface is uniquely determined by its metric and mean curvature function. It is known that this is the case for generic surfaces, and also for topological spheres. We explicitly construct a pair of immersed tori in three dimensional Euclidean space that are related by a mean curvature preserving isometry. These tori are the first examples of compact Bonnet pairs. Moreover, we prove these isometric tori are real analytic. This resolves also a second longstanding open problem on whether real analyticity of the metric already determines a unique compact immersion. Discrete differential geometry is used to find crucial geometric properties of surfaces. This is a joint work with Tim Hoffmann and Andrew Sageman-Furnas.

Tuesday 2

Speaker: Tristan Rivière (ETH)

Title: Hamiltonian Stationary Surfaces in Sasakian geometry. Variational approaches.

Abstract: In the early 90's Yong-Geun Oh introduced the problem of studying critical points of the area among Lagrangian surfaces in a symplectic riemannian manifold. Such surfaces are called Hamiltonian stationary or sometimes H-minimal surfaces. This variational problem is motivated by natural questions such as the study of the Plateau problem in Lagrangian homology classes, the construction of calibrated minimal surfaces in Calabi Yau geometry (Thomas-Yau conjecture) or even the minmax constructions of minimal surfaces in spheres. We will first present the difficulties of dealing with the Hamiltonian stationary equation in general and present as a "warning" the construction of "pathological solutions" to this equation in 2 dimension which are nowhere continuous. Then we will turn to the special case of area minimizing H-minimal surfaces and the discovery in the early 2000 of a family of singularities of conical type by Schoen and Wolfson around which the Maslov class realized by the Lagrangian planes is non trivial. We will raise the question of the possible location of these singularities and whether they "interact" or not. In relation with this question, I will mention a direct method for constructing Hamiltonian stationary discs with prescribed Schoen Wolfson cones (joint work with Filippo Gaia and Gerard Orriols). In the second part of the talk we will prove that every non trivial minmax operation in a closed 5 Sasakian manifold is realised by a weak GMT/PDE version of Hamiltonian Stationary surfaces. We conjecture that these surfaces have only isolated SW cones and isolated branched points. We will mention the very recent progresses in the direction of proving this conjecture.

Speaker: Lucas Ambrozio (IMPA, Brazil)

Title: Generalizations of Zoll surfaces in minimal submanifold theory.

Abstract: A Zoll surface is a Riemannian surface homeomorphic to a sphere whose non-trivial geodesics are all closed and have the same length. As surprising as it may seem, there are Zoll surfaces that are not the Euclidean sphere. And while these surfaces have been known for more than a hundred years, their full classification remains elusive.

However, Zoll surfaces are not only interesting geometric objects in themselves. We will discuss two variational problems where Zoll surfaces appear to be central objects to be understood. This will motivate the introduction of generalised notions of Zoll surfaces in the context of minimal submanifold theory, where similar variational problems are meaningful and have only recently started to be investigated. Finally, we will discuss the recent progress

done towards the development of a general theory of higher dimensional Zoll-like objects.

Wednesday 3

Speaker: Giada Franz (MIT)

Title: Construction and properties of free boundary minimal surfaces via min-max.

Abstract: A free boundary minimal surface (FBMS) in a three-dimensional Riemannian manifold is a critical point of the area functional with respect to variations that constrain its boundary to the boundary of the ambient manifold. It is natural to ask about the existence of FBMS (in a given ambient manifold) and their properties (topology, area, Morse index, etc.).

In this talk, we will analyze these questions through the lens of Simon-Smith variant of Almgren-Pitts min-max theory. More precisely, we will see how this method allows the construction of FBMS with prescribed properties (symmetry, topology, Morse index, etc.), by presenting new developments and discussing the limits and perspectives of this approach.

Speaker: Paul Laurain (Université Paris Cité)

Title: Morse Index stability for Yang-Mills fields.

Abstract: After introducing the Yang-Mills setting, I will review some classical works about Yang-Mills in critical dimension (dimension 4) such as Uhlenbeck's gauge extraction and quantization of the energy. Finally, I will show how, with M. Gaurvit, we have proved the upper and lower semi-continuity for the index of sequences of Yang-Mills fields.

Thursday 4

Speaker: Nicolaos Kapouleas (Brown University)

Title: Recent results on minimal (hyper)surface doublings and desingularizations.

Abstract: As an introduction I will recall earlier results on gluing constructions of minimal surfaces by PDE methods. I will concentrate then on recent results (in collaboration with Jiahua Zou) and ongoing work.

Speaker: David Wiygul (Università degli studi di Trento)

Title: Existence and uniqueness of desingularizations of intersecting Clifford tori.

Abstract: M. Soret and J. Choe constructed (in 2013) two infinite families of closed embedded minimal surfaces in the round 3-sphere that can be interpreted as desingularizations of collections of intersecting Clifford tori. N. Kapouleas and I constructed further examples admitting the same interpretation and having high genus. In this talk I will present some new existence and uniqueness results for such surfaces, obtained in recent and ongoing work with Kapouleas.

Friday 5

Speaker: Reto Buzano (Università degli studi di Torino)

Title: Mean curvature flow and mean-convex embeddings.

Abstract: In this lecture we will study the space of mean-convex embeddings of spheres and tori into three-dimensional manifolds. After motivating the problem, we first start with a very brief and intuitive overview of mean curvature flow with surgery and explain a gluing construction to topologically undo the surgeries again. We then use this surgery and gluing approach to prove that the moduli space of mean-convex embedded two-spheres in \mathbb{R}^3 is path-connected, while the connected components of the moduli space of mean-convex embedded tori in \mathbb{R}^3 are in bijective correspondence with the knot classes associated to the embeddings. Next, we generalise to non-Euclidean ambient manifolds and prove that the moduli space of mean convex two-spheres embedded in complete, orientable 3-dimensional manifolds with nonnegative Ricci curvature is path-connected. This result is sharp in the sense that neither of the conditions of (strict) mean convexity, completeness, and nonnegativity of the Ricci curvature can be dropped or weakened. We finally study the number of path components of mean convex Heegaard tori in ambient manifolds with nonnegative Ricci curvature. We prove that there are always either one or two path components and this number does not only depend on the homotopy type of the ambient manifold. We give a precise characterisation of the two cases and also discuss what happens if the mean convexity condition is weakened to nonnegative mean curvature. This is joint work with Haslhofer-Hershkovits, and with Maillot.

Speaker: Gerard Besson (Université Grenoble)

Title: Contractible 3-manifolds and Positive Scalar Curvature (After Jian Wang)

Abstract: I will describe the work of my former student Jian Wang on

positive scalar curvature and some open 3-manifold. We will look at the simplest example, the so-called Whitehead manifold, which I will describe, and show that it does not carry any complete metric of non negative scalar curvature. The proof uses extensively properties of some minimal disks. I will then discuss a couple of questions and projects in dimension 3 and 4.